Cosmological Perturbations: Myths and Facts

Paul J. Steinhardt[1,2]
[1] *Department of Physics, Princeton University, Princeton, NJ 08544, USA*
[2] *Institute for Advanced Study, Princeton NJ 08540 USA*

We discuss (and debunk) ten common myths about cosmological perturbations in inflationary and ekpyrotic/cyclic models and their implications for future observations.

The purpose of this keynote address is to discuss the predictions of density perturbations in cosmological models. For astrophysicists, density perturbations provide one of the best means for discriminating competing models. For particle physicists and string theorists, they can provide qualitative and quantitative information about microphysics that is not accessible in accelerators.

Inflation [1] was shown to generate a nearly scale-invariant, adiabatic, gaussian spectrum of density perturbations over twenty years ago [2]. Yet, revisiting the subject is worthwhile for several reasons. First, there have been a number of misconceptions that have developed over the years that ought to be clarified now that their properties are actually being measured. Second, a new mechanism for generating the same kind of density perturbation spectrum has been recently discovered in ekpyrotic [3] and cyclic [4] models, and it useful to compare the old and new approaches. Here, also, some misconceptions have already developed that we shall address. Third, a new "duality" relating perturbation spectra in inflationary and ekpyrotic models has been found [5, 6]. These recent theoretical advances, combined with the availability of highly precise measurements of the density perturbation spectrum [7], make this discussion especially timely.

Myth 1: Inflation makes no firm predictions

We will argue below that inflation and the cyclic model make firm predictions for the characteristic mass scale and equation of state during inflation, the spectral tilt of the scalar perturbations and the gravitational wave amplitude. This conclusion appears to run contrary to many papers on inflation that suggest the predictions can vary over many orders of magnitude. Some go so far as to suggest that inflation makes no firm predictions and is not falsifiable.

The disagreement is simple to explain. First, we begin here with the idea that inflation is a powerful explanatory model. Seeking the most powerfully explanatory model, we identify the *minimal* conditions and minimal tuning required to meet its goals – to solve the horizon, flatness and monopole problems and to generate an acceptable spectrum of density perturbations. From these minimal conditions, we derive rather definitive predictions.

For the past twenty years, though, theorists have made a small industry out of constructing increasingly complicated models of inflation, adding further ingredients (masses, fields, symmetries, *etc.*) that change the predictions. How are we supposed to regard these revised predictions? As with any scientific theory, one should recognize that the changed predictions are not natural to inflation itself, but, rather, hinge on the justification of the added ingredients. Since their addition pushes inflation into a small corner of what it naturally allows, one might even say that there is a certain tension between the ingredients and inflation.

A second reason why many surveys of inflationary models claim a broader range of predictions is because they allow extra parameters, fields, tunings, *etc.* without penalty. This type of analysis does not reflect good scientific common sense that favors the fewest degrees of freedom and the fewest tunings. Adding to the confusion is the fact that the inflationary analyses do not always make the extra parameters, fields and tunings apparent. For example, some analyses consider the shape of the inflaton potential over the range where perturbations leave the horizon without considering how this links up with the part of the potential required for inflation to end. The statistical treatment is as if the two parts can be adjusted independently so that all combinations are equally probable. In practice, for most choices, the two parts tend to interfere, requiring extra fine-tuning to fit the them together. The surveys treat models with and without extra tuning with equal weight.

Here we try to present an approach that gives models the weight that makes good scientific common sense. The reason for expounding on this issue is because it affects the current outlook for testing inflationary cosmology. Following the natural inflationary predictions, we will see the critical tests of the inflationary paradigm are within reach of experiment in the next few years. If inflation passes those tests, it is important to recognize that the predictions are meaningful and an important milestone has been achieved.

We already have a working example. Inflation naturally predicts a flat universe. Yet, numerous authors explored inflationary models with extra tunings or fields arranged to obtain an open, subcritical universe. This effort reached its zenith in the mid-1990s when it became apparent that the mass density is less than the critical density and the curvature of the universe was in doubt. At that time, some argued that open inflationary was as plausible as flat inflation. They often argued that, since inflation requires some degree of fine-tuning anyway, additional fine-tuning should be viewed as disadvantageous. Others (such as myself) argued that an open universe was incompatible with inflation because the extra fine-tuning was introduced only to push inflation towards a prediction which was otherwise unlikely. Observations have shown that the universe is flat. These observations are now embraced as evidence for inflation (by some of the same people who propounded Open Inflation). You cannot have it both ways you cannot say that inflation allows all possibilities and then claim success when the universe is proven to be flat. If you believed inflation made no prediction, it is not success. However, if you are cautious to discriminate between what inflation naturally predicts versus what happens when you add extra ingredients, then you can rightfully claim triumphant agreement between the simplest theory and observation. This same argument applies to other predictions we describe below.

Simplicity is a guiding principle that motivates inflation, and so one should place great weight on predictions obtained by assuming the fewest number of elements and mass scales. This, we will show, leads to definite, testable predictions for near-future experiments. At this point, there is every reason to be optimistic that the natural predictions will be verified empirically.

Myth 2: Inflation is required for quantum fluctuations on subhorizon scales to exit the horizon

Current measurements [7] suggest that the spectrum of density perturbations is nearly scale-invariant, adiabatic and gaussian, and so we will restrict ourselves to models that predict these properties. These models have a common feature: the origin of the density perturbations are quantum fluctuations on sub-horizon scales that exit the horizon in one phase and re-enter the horizon in the usual radiation or matter dominated epoch. In using concepts like entering and exiting the horizon, we are always referring to formulations of the microphysics in the Einstein frame for which the gravitational action has standard Einstein-Hilbert form:

$$S = \int d^4x\sqrt{-g}\left[\frac{1}{2}R + \ldots\right] \tag{1}$$

where $g \equiv det(g_{\mu\nu})$ is the determinant of the metric, R is the Ricci scalar and reduced Planck units $8\pi G = 1$ are used.

The key parameter in describing the shape of the density perturbation spectrum is the equation of state w during the phase when the quantum fluctuations exit the horizon, where w is the ratio of the pressure p to the energy density ρ. We will treat the stress-energy as a scalar field with self-interaction potential $V(\phi)$. The pressure for a homogeneous scalar field is, then, $p = \frac{1}{2}\dot{\phi}^2 - V$, and the energy density is $\rho = \frac{1}{2}\dot{\phi}^2 + V$, where dot represents the time derivative. For a Friedmann-Robertson-Walker metric $ds^2 = -dt^2 + a^2 d\mathbf{x}^2$ and Hubble parameter $H \equiv \dot{a}/a$, the continuity equation

$$\dot{\rho} = -3H(p+\rho) \tag{2}$$

has solution

$$\rho \sim \frac{1}{a^{3(1+w)}} = \frac{1}{a^{2\epsilon}} \tag{3}$$

where

$$\epsilon \equiv \frac{3}{2}(1+w). \tag{4}$$

The parameter ϵ is one of the well-known "slow-roll" expansion parameters, which is small in inflationary models because $1 + w \ll 1$. Here we will consider general w; ϵ should not be assumed to be small unless explicitly specified to be so. Substituting (3) into the Friedmann equation, we find that the scale factor is

$$a \sim t^{2/3(1+w)} = t^{1/\epsilon} \tag{5}$$

and the Hubble radius is

$$H^{-1} \sim a^{\epsilon}. \tag{6}$$

Note that we will use $\sim$ throughout to represent "approximately proportional to." This paper is intended as a pedagogical introduction and, for this purpose, factors of order unity will be dropped except in cases where they are important.

From recent developments [3–5, 8, 9], we now understand that there are two completely different approaches for causing quantum fluctuations on sub-horizon scales to exit the horizon. The first approach occurs if the universe is expanding. Then, a mode can exit the horizon if it is stretched at a rate faster than the Hubble radius. In other words, $a(t)$ must grows faster than $H^{-1}(t)$. From (5) and (6), this requires $\epsilon < 1$.

The alternative approach arises in a contracting universe. Quantum fluctuations exit the horizon if $a(t)$ shrinks more slowly than the $H^{-1}(t)$. Using the same relations, (5) and (6), it is easy to see that this condition occurs if $\epsilon > 1$.

The expanding universe with $\epsilon \ll 1$ and the contracting universe with $\epsilon \gg 1$ correspond to the inflationary and ekpyrotic regimes, respectively. (The term "ekpyrotic" was first used [3] to describe a model which began with a contraction expansion phase where $\epsilon \geq 1$. The cyclic model [4] has repeated periods of expansion and contraction; the contraction phase is essentially identical to the ekpyrotic case.) We will first focus on the expanding, inflationary case that is already familiar to many. After analyzing its properties, we will turn to the contracting case and discuss the relation between them.

Myth 3: The mass or energy scale for inflation is poorly determined

The derivation of the density perturbation spectrum in inflationary models can be found in many standard reviews. The result for the Fourier mode with physical wavenumber k is

$$\frac{\delta\rho}{\rho}(k) \sim \frac{H^2}{\dot{\phi}} \tag{7}$$

where the expression on the rhs is to be evaluated when the physical wavelength is equal to the Hubble radius during inflation. The shape and amplitude of the spectrum near wavenumber k depend on three time-dependent parameters

- the equation of state w as mode k leaves the horizon,
- the number of e-folds of inflation, N, between when the mode exits and inflation ends, and
- the characteristic energy scale during inflation, $M \sim \rho^{1/4}$; the inflaton potential energy density is M^4.

We now consider the conditions these parameters must satisfy to resolve various cosmological problems.

Resolving the horizon problem: The horizon problem is resolved by having a sufficient number of e-folds N_{total} such that the current horizon radius H_0^{-1} lay within a causal horizon radius H_I^{-1} before inflation begins. Extrapolating back in time, the current radius was smaller by a factor of $a_{end}/a_0 = T_0/T_{end}$ at the *end* of inflation, where T is the temperature. Further extrapolating to the *beginning* of inflation, the radius is reduced by an additional $e^{-N_{total}}$. The resolution of the horizon problem translates into the constraint

$$H_0^{-1} \cdot \left(\frac{T_0}{T_{end}}\right) \cdot e^{-N_{total}} < H_I^{-1} \tag{8}$$

or, using the fact that $H \sim T^2$,

$$N_{total} > \ln \frac{T_{end}}{T_0} \sim 60, \tag{9}$$

where we have used $T_{end} \sim 10^{15}$ GeV and $T_0 = 3\,K \sim 10^{-13}$ GeV.

We will use N_{total} to represent the total number of e-folds of inflation; the symbol $\bar{N} \leq N_{total}$ to represent the number of e-folds before the end of inflation when the mode on the horizon today exited the horizon during inflation; and N (without the bar) to represent the time-like variable that runs from N_{total} to zero as inflation proceeds from beginning to end. Our computation above has shown that $\bar{N} \sim 60$.

Perturbation amplitude: The density perturbation amplitude for $\bar{N} \sim 60$ must satisfy

$$\frac{\delta\rho}{\rho} \sim \frac{H^2}{\dot{\phi}} \sim \frac{\rho}{\sqrt{\rho + p}} \sim \sqrt{\frac{\rho}{1+w}} \sim 10^{-5}. \tag{10}$$

This places the constraint on the height of the inflaton potential V or, equivalently, the characteristic energy scale during inflation, $M \sim \rho^{1/4} \sim V^{1/4}$:

$$M \sim 10^{-5/2}\,(1+w)^{1/4}. \tag{11}$$

Notice the weak dependence on $1+w$; its value is much less than unity during the inflationary phase, but, assuming that it is not absurdly small, we obtain

$$M \sim 10^{-3}. \tag{12}$$

This estimate for M should be regarded as a prediction of inflation, as sure as any other. Yet, this remark seems confusing in light of the many papers which treat this scale as a free parameter. Some have even argued that a much smaller scale, $M \sim 10^{-6}$, is more natural based on consideration of supersymmetry.

We can see now that the only way to obtain $M \sim 10^{-6}$, say, is if $1+w$ is incredibly fine-tuned: $1+w = 10^{-14}$. This is 12 orders of magnitude of tuning beyond what is required to resolve the horizon, flatness, and monopole problems.

For some, w is an unfamiliar parameter and it may not be apparent that the condition $1+w = 10^{-14}$ really constitutes fine-tuning. They note that numerous authors have constructed inflation potentials $V(\phi)$ with $M \sim 10^{-6}$ (and smaller). Where is the additional tuning? Inflationary model-builders are very clever, but the extra tuning is always there. Let us translate our constraint on w into a constraint on the inflaton potential V and its derivatives with respect to ϕ (indicated by primes). In the inflationary limit where $\frac{1}{2}\dot{\phi}^2 \ll V$,

$$1+w \sim \frac{\dot{\phi}^2}{V} \sim \left(\frac{V'}{V}\right)^2 \tag{13}$$

where we have used the equation of motion for ϕ

$$\ddot{\phi} + 3H\dot{\phi} = -V' \tag{14}$$

and taken the slow roll approximation $\ddot{\phi} \ll 3H\dot{\phi}$. If H is slowly varying and $V'(\phi_N) \ll V''\phi_N$, then it is straightforward to show (using the equation of motion) that the number of e-folds before the end of inflation is

$$\bar{N} \sim \frac{V}{V''} = \frac{V}{V'}\frac{V'}{V''}, \tag{15}$$

where we evaluate the expression for the mode whose wavelength is equal to the Hubble horizon today, $\bar{N} \sim 60$. The expression for $\bar{N}$ has been expanded in terms of two parameters with the dimensions of mass which we might call $m \equiv V/V'$ and $m' = V'/V''$. Eq. (15) implies

$$m\,m' = \bar{N}$$

and (13) implies $m^2 = (1+w)^{-1}$. Putting these together, the simplest possibility is to assume a single dimensionful mass scale, or $m \sim m' \sim \sqrt{\bar{N}}$, in which case one obtains $1+w = 1/\bar{N}$. We will see below that this value of w satisfies all the constraints of inflation. But, then, of course, we are forced by (11) to have $M \sim 10^{-3}$. Now, suppose we insist on $M \sim 10^{-6}$ and $1+w = 10^{-14}$. Then, $m = 10^7$ and $m' = \bar{N} \times 10^{-7}$. We see from this argument that fine-tuning w is equivalent to introducing a new hierarchy problem – that is, a ratio of m/m' that is far from unity – a hierarchy that is not required by inflation.

Myth 4: The inflationary perturbation spectrum is equally likely to be red or blue.

Further constraints fix the spectral tilt of the inflationary perturbation spectrum, causing it to be red.
Tilt: We can express the density perturbation spectrum in (10) as

$$\frac{\delta\rho}{\rho} \sim \sqrt{\frac{\rho}{1+w}} \sim A k^{(n_s-1)/2}, \tag{16}$$

where A is constant and n_s is the spectral index. The deviation from unity is called the spectral tilt. In general, n_s may be scale dependent, but we imagine examining a small enough range of k about k_N (the mode that leaves the horizon N e-folds before the end of inflation) that n_s is approximately constant. Then, using (3) and (4) plus the relation $dN = -d\ln k = -d\ln a$, we obtain

$$n_s - 1 = \frac{d\ln\sqrt{\rho/(1+\mathrm{w})}}{d\ln k} = -2\epsilon + \frac{d\ln\epsilon}{dN} \tag{17}$$

Recall that $0 < \epsilon \ll 1$ and that ϵ must increase as inflation ends (or, equivalently, N becomes small). Hence, both terms in (17) are negative and n_s must be less than unity ("red") as inflation comes to a close. The only way to make a blue spectrum during the last 60 e-folds of inflation is to introduce a finite range of the potential along which epsilon decreases rapidly; this must end after a few e-folds in order to allow inflation to complete after 60 e-folds. Extra tunings and/or fields must be introduced to accomplish this effect. There is no reason from the point-of-view of inflation alone for there to be the extra ingredients.
Duration of inflation: We are interested in the values of n_s, w, and M when the modes k left the horizon during inflation with N e-folds of inflation remaining. If δt_N is the time interval for the last N e-folds, then the constraint that inflation ends is that H should change substantially by the end of this time interval, or, equivalently,

$$\dot{H}\Delta t_N \sim H. \tag{18}$$

Since $N \equiv H\Delta t_N$, we have $N \sim H^2/\dot{H}$. According to the Friedmann equations, $H^2 = \frac{1}{3}\rho$ and $\dot{H} = \frac{1}{2}(\rho+p)$. Hence, we have

$$N \sim \frac{2}{3}\frac{\rho}{\rho+p} = \frac{2}{3(1+w)} = \frac{1}{\epsilon(N)}. \tag{19}$$

By imposing the constraint that inflation ends after N e-folds, we have obtained an expression relating ϵ and N.
Tilt revisited: Substituting our expression for $\epsilon(N)$ in (19) into our expression for the tilt in (17), we obtain

$$n_s - 1 \sim -\frac{3}{N}. \tag{20}$$

As anticipated above, the prediction is that the spectrum is red. Furthermore, if we evaluate the expression on the current horizon scale, corresponding to $\bar{N} \sim 60$, the spectral index is predicted to be $n_s \approx 0.95$. All the $\sim$'s mean that there is some uncertainty in this estimate which adds up to roughly 2 or 3 per cent. Part of the uncertainty might appear to be due to the uncertainty in $\bar{N}$. However, this uncertainty can be removed by combining our observation of the amplitude on the current horizon scale with our estimate for ϵ in (19) to obtain one condition on $\bar{N}$:

$$\frac{\delta\rho}{\rho} \sim \sqrt{\frac{\rho}{1+w}} \sim M^2\,\epsilon^{-1/2} \sim M^2\,\bar{N}^{1/2}. \tag{21}$$

Our earlier condition on $\bar{N}$ in (9) was expressed in terms of T_{end} which is $\sim M$. Combining the two, we find that $T_{end} \sim M \sim 10^{16}$ GeV and $\bar{N} \sim 60$. This is consistent with our estimates all along and suggests that there is little room to play with N without introducing extra fields and parameters.

Myth 5: The detectability of tensor perturbations in the cosmic microwave background depends sensitively on magnitude of the inflaton potential

Gravitational Wave Spectrum: Inflation produces a nearly scale invariant spectrum of tensor perturbations as well as scalar perturbations. The mean square amplitude is $\mathcal{T} \sim V \sim M^4$, where V is the magnitude of the inflaton potential when the modes leave the horizon during inflation. For $M \sim 10^{-3}$, the tensor contribution to the microwave background anisotropy is of order a few microkelvin. Near-future experiments aiming to refine the measurements of the temperature anisotropy and to detect the B-mode polarization of the microwave background should exceed the sensitivity to detect the gravitational waves [10, 11] by an order of magnitude or more [13].

However, some point out that $\mathcal{T}$ is proportional to M^4 and that M is not well constrained. A decrease in M by an order of magnitude or more reduces the tensor contribution to the point where it is very difficult to detect. Hence, it is argued, the microwave background anisotropy and polarization test only a narrow range of parameters.

The analysis is somewhat misleading because it focuses on $\mathcal{T}$ alone without considering the mean square scalar fluctuation amplitude, $\mathcal{S} \sim (\delta\rho/\rho)^2$. In fact, both $\mathcal{S}$ and $\mathcal{T}$ are proportional to M^4 and their sum is constrained to equal the observed amplitude on large angular scales [12]. A better way to judge the detectability of tensor fluctuations is to consider the ratio

$$r \equiv \frac{\mathcal{T}}{\mathcal{S}} \sim \frac{H_I^2}{H_I^4/\dot{\phi}^2} \sim \frac{\rho+p}{\rho} = B\epsilon \tag{22}$$

where B is a constant. The precise value of B depends on the multipole moment or combination of multipole moments for which one measures $\mathcal{S}$ and $\mathcal{T}$. Using the

WMAP convention, the value is $B \sim 14$. Notice that the height of the inflaton potential (or M) drops out completely from this ratio. Hence, if the scalar fluctuations have been detected, the detectability of the tensor fluctuations only depends on the equation of state parameter, $\epsilon = \frac{3}{2}(1+w)$.

Myth 6: The gravitational wave spectrum in inflation is likely to be undetectable.

From (9), we obtain the prediction [5]

$$r \sim \frac{14}{N} \approx 23\%. \tag{23}$$

(The WMAP value of $r_{WMAP}a$, which is based on a different normalization convention, is 1.16 times this value.) This value is quite substantial, perhaps within the range of a full WMAP survey of the temperature anisotropy and probably within the range of planned polarization experiments. There is some uncertainty in this prediction, but it is fair to say that pushing r below 1% or above 50% requires additional fine-tuning beyond what is required to solve the canonical cosmological problems.

Hence, the expectation is that, if inflation is right, a detection of primordial gravitational waves is around the corner. This should be regarded as a strong prediction of inflation to be celebrated if verified, just like the prediction of spatial flatness. The suggestion that the prediction can be easily evaded improperly reduces the significance of the detection (or the significance of a failed detection).

In particular, consider the plot of the r-n_s plane shown in Fig. 1, an adaptation of a figure constructed to show the combined constraints of WMAP and SDSS data [17]. The contours indicate the regions excluded by WMAP alone or WMAP combined with SDSS. The dark region represents the natural predictions of inflation, where here we have including uncertainties in the estimates above. The current situation is very exciting. Half of the likely region is excluded, and half is allowed.

A more sensitive probe is measuring the B-mode polarization of the microwave background. Planck may probe down to $r \sim 10^{-3}$, and a dedicated polarization satellite a decade or so away might get down as far $r \sim 10^{-6}$ [13]. The point is that these experiments cover a wide span that generously encompasses the dark region in Fig. 1.

Fig. 1 leads one to view the entire r-n_s plane as being equally likely, whereas our point is that the situation is really different: the dark region is what inflation naturally predicts and the rest of the plane requires additional fine-tuning. Yet, Fig. 2 illustrates a common way of dividing up the r-n_s plane into subregions representing different types of inflationary models. How does this standard subdivision into types relate to the claim about fine-tuning? Examining each type, one finds a different kind of fine-tuning. In the case of hybrid inflation, this type model is characterized by having two or more

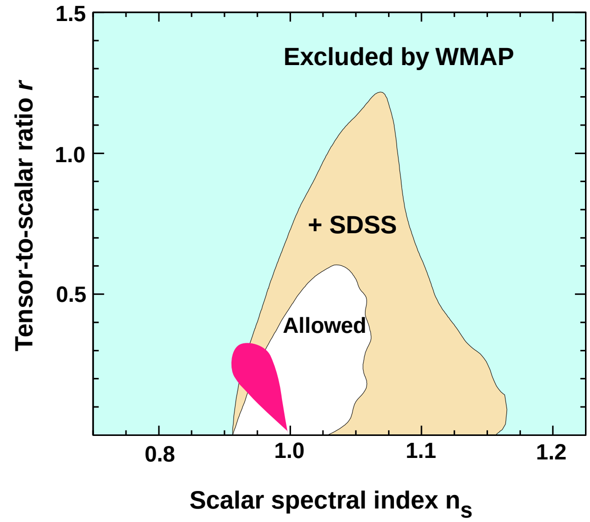

FIG. 1: Plot showing WMAP and SDSS constraints on the scalar spectral index n_s versus the ratio of tensor to scalar fluctuations r (adapted from Tegmark *et al.*, astro-ph/0310723).

scalar fields in the inflaton sector, each playing an active role. If we consider two-field inflation models in general with a minimum of tuning, these will give predictions in the dark zone on Fig. 2. To get outside the zone requires tuning the masses and couplings of the fields to be orders of magnitude different from one another – there lies the extra tuning. Small field inflationary models often entail a single scalar field, but they correspond to choosing $m/m' \gg 1$ in Eq. (15). Large field inflationary models correspond to $m/m' \ll 1$. In other words, the brands of inflation outside the dark zone each correspond to introducing an additional fine-tuning of one sort or another.

A. Myth 7: Inflation is the only mechanism for generating nearly scale-invariant, adiabatic, Gaussian perturbations.

We have argued that there are two kinds of cosmological backgrounds in which quantum fluctuations can exit the horizon: an accelerating expanding universe with $\epsilon < 1$ and a slowly contracting universe with $\epsilon > 1$. Assuming that the equation of state is not changing too rapidly, the fluctuations of the Newtonian potential (in Newtonian gauge) for either the expanding or contracting case can be computed. It is straightforward to obtain a general expression for the spectral index as a function of ϵ [9, 18]:

$$n_s - 1 = -\frac{2}{(1-\epsilon)^2}\left[\epsilon - \frac{1-\epsilon^2}{2}\frac{d\ln\epsilon}{dN}\right] \tag{24}$$

Examining this expression, one finds that a nearly scale-invariant spectrum ($n_s \approx 1$) can be obtained in two limits: $\epsilon \ll 1$ and $\epsilon \gg 1$. The first limit corresponds to

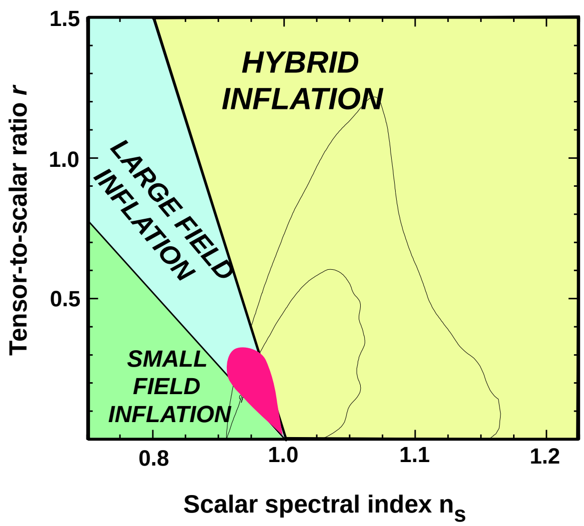

FIG. 2: Plot showing n_s versus r with a conventional subdivision of the plane into different types of inflation (adapted from Tegmark *et al.*, astro-ph/0310723).

inflation, where we obtain for $\epsilon \ll 1$

$$n_s - 1 = -2\epsilon + \frac{d\ln\epsilon}{dN} \tag{25}$$

and the second limit corresponds to the ekpyrotic/cyclic models, where we obtain for $\epsilon \gg 1$

$$n_s - 1 = -\frac{2}{\epsilon} - \frac{d\ln\epsilon}{dN}. \tag{26}$$

Hence, inflation is not the unique mechanism for generating nearly scale-invariant, gaussian, adiabatic perturbations. In fact, the inflating and ekpyrotic solutions are related by a surprising "duality" symmetry [5, 6]. The expressions for n_s transform into one another if $\epsilon \to 1/\epsilon$. That is, for every inflationary model with a given ϵ, there is an ekpyrotic model with $\epsilon' \equiv 1/\epsilon$ which produces a density perturbation spectrum with precisely the same tilt. The relative evolution of the scale factor a and the Hubble radius H^{-1} is the same in the two models. Inflation corresponds to a growing rapidly compared to H^{-1}; and, ekpyrosis corresponds to a shrinking less rapidly than H^{-1}; but the relative change is the same if $\epsilon \to 1/\epsilon$.

The duality is important for two reasons. First, it makes clear that there is nothing so mysterious about the ekpyrotic mechanism for generating density perturbations. We can see that the calculation is conceptually and quantitatively a straightforward generalization of the well-known inflationary analysis. Secondly, the duality makes it clear that measurements of the scalar spectral index cannot be used to determine if the density fluctuations were produced in an inflationary or ekpyrotic phase. The predictions at linear order for the two models are not merely similar, but precisely equivalent under the dual transformation. (Non-linear corrections are different.)

The analysis assumes that the modes re-enter the horizon at a later stage with the same amplitude with which they left. In the case of inflation, re-entry is made possible by ending inflation and entering a slower expansion phase with $\epsilon > 1$ during which H^{-1} grows faster than a. In the ekpyrotic/cyclic case, a bounce to an expanding phase and a change from $\epsilon > 1$ to $\epsilon < 1$ is required.

In ekpyrotic and cyclic models, the contraction phase and bounce correspond to a collision between two orbifold planes. The contracting phase commences after the universe has undergone a period of accelerated expansion. The temperature and density are negligible after the accelerated expansion period and remain so up to the point that the branes collide. The equation of state is $\epsilon \geq 1$ during the contracting phase with $\epsilon \to 1$ at collision. In the effective 4d moduli space field theory, the 4d effective scale factor approaches zero at collision.

Myth 8: In cyclic models, the density perturbation spectrum that emerges after the bounce is blue rather than scale-invariant.

The 4d calculations of perturbations in ekpyrotic/cyclic models has been controversial and have led to an ambiguous conclusion. An issue has been that the gauge invariant variables Φ (the Newtonian potential perturbation) and ζ (the curvature fluctuation on comoving hypersurfaces) cannot both be continuous at the bounce. More generally, two independent linear combinations of ζ and Φ cannot be both continuous at the bounce [14]. If one is continuous, the other must jump. The calculation depends critically on which linear combination ought to match continuously, and the right choice is ambiguous. (In inflationary models, both Φ and ζ are continuous, so there is no ambiguity.)

Some suggested that ζ is the proper variable to match [15]. During the contracting phase, Φ obtains a nearly scale invariant spectrum of fluctuations, but ζ obtains a blue one. Hence, if ζ is the appropriate variable to match, the perturbation spectrum after the bounce is blue, rather than scale-invariant. This is the source of the myth.

On the other hand, if the appropriate variable to match is some linear combination of ζ and Φ, a linear combination of blue and scale-invariant spectra would emerge after the bounce. At long wavelengths, the blue contribution is negligible and the spectrum is effectively scale-invariant.

Hence, the ambiguity in matching conditions makes uncertain whether a scale-invariant spectrum emerges after the bounce. Although different physical arguments were raised to suggest one combination or the other, the bottom line after considerable exchanges is that a detailed understanding of the bounce itself is required to obtain an unambiguous answer.

To resolve the ambiguity in ekpyrotic/cyclic models, Tolley *et al.* [16] considered the bounce as a collision of branes in 5d. The 4d moduli theory is used to determine the conditions on the branes approaching the bounce; then 5d general relativity is used to follow the propagation of bulk perturbations through the bounce. The fact that the branes exist before, during and after the bounce imposes sufficient conditions to make the matching condition unique.

If the extra dimension is orbifolded, the bulk and branes are distinguished and there is no 5d analogue of ζ since there are no 5d comoving (uniform) hypersurfaces. Nevertheless, ζ_{4d} has a natural interpretation in the 5d theory. There is on each 4d brane a curvature perturbation variable $\zeta_\pm$ which is conserved for modes outside the horizon (just like ζ_{4d}). One can choose hypersurfaces in the 5d theory where $\zeta_+ = \zeta_- = \zeta_{4d}$. This corresponds to a slicing where ϕ is uniform, where ϕ is the moduli field which determines the distance between branes. Naively, the distance is $d = \ln(-\coth\phi)$. Hence, it is tempting to conclude that the bounce at $\phi \to -\infty$ corresponds to a surface of constant $\zeta_\pm = \zeta_{4d}$ and that ζ_{4d} is the appropriate variable to match.

However, Tolley *et al.* [16] show that the correct variable to match is ζ plus a correction proportional to v_{coll}^4, where v_{coll} is the relative velocity of the colliding branes. The collision does not occur precisely on a surface of constant ζ or constant ϕ. As the branes approach, a scale-invariant spectrum of (massive) bulk modes is excited by the motion of the branes with scale-invariant ripples, which produces a small non-zero warping depending on the relative velocity of branes near the collision, v_{coll}. These fluctuations in the y direction correspond to fluctuations in the distance between branes. That is, even though $\zeta_\pm$ and ϕ are uniform, the distance between branes fluctuates. Hence, the collision does not occur simultaneously on any slice of constant ζ. To match perturbations, a small adjustment has to be made to a different slicing where the bounce is simultaneous. Then, the variable that matches continuously is not ζ; and ζ itself must jump by an amount that depends on the nearly scale-invariant bulk fluctuations. ζ, thereby, obtains a scale-invariant spectrum resulting in a scale-invariant density perturbation spectrum after reheating.

The discrepancy between the uniform bounce and uniform ζ slices is proportional to v_{coll}^4, which is small for non-relativistic velocities. Consequently, scale-invariant fluctuations produced in the contracting phase pass through the bounce, but the amplitude is small. Hence, the ekpyrotic mechanism leads naturally to small amplitudes (whereas inflation leads naturally to large amplitudes and must be tuned to obtain small amplitudes).

Another way to interpret the result is that the bulk warp fluctuations induce a small correction to the formula relating the interbrane distance to the 4d moduli field ϕ, $d = \ln(-\coth\phi) + v_{coll}^4 f(\nabla\phi, \phi)$. Note that f is not a function of ϕ alone; otherwise, the distance would be uniform on constant ϕ. In the 4d theory, the result means that $a \to 0$ does not coincide precisely with $\phi \to -\infty$. Rather, there is a correction dependent on v_{coll} and gradients of ϕ. Rather than compute the precise form of f, Tolley *et al.* [16] find it is easier to match across the bounce using the 5d matching construction.

Myth 9: The cyclic and inflation models cannot be distinguished observationally

Although the scalar perturbation spectra in inflationary and cyclic models are indistinguishable to leading order in perturbation theory due to the duality that relates a theory of one type to the other, there are other differences between the two theories. These can be traced back to the fact that inflation corresponds to $\epsilon \ll 1$ and ekpyrotic/cyclic corresponds to $\epsilon \gg 1$. The first case corresponds to ultra-rapid accelerated expansion in which the Hubble parameter is large compared to the curvature of the scalar field effective potential, V. The second case corresponds to ultra-slow contraction in which the Hubble parameter is 10^{100} times smaller than in inflation and negligible compared to the curvature of V.

In inflation, the large value of H compared to V'' means that the equation of motion for the inflaton is nearly the same as for a massless field. The metric components satisfy precisely the massless equation of motion. Hence, the inflaton and metric components both obtain the same spectrum of fluctuations.

In the ekpyrotic/cyclic model, where the Hubble parameter is negligibly small, the fluctuations in the scalar field depend critically on V and its derivatives. However, the metric components are massless and do not depend directly on V. In fact, since the background is very slowly contracting, the metric fluctuations are blue, roughly what is to expected in a nearly static background [3, 8, 19, 20]. The difference between scalar and tensor equations of motion accounts for why the scalar spectrum is nearly scale invariant and the tensor spectrum is very blue.

The comparisons of the inflationary and ekpyrotic/cyclic predictions have taught us that the shape of the tensor fluctuation spectrum provides direct information about the conditions under which the scalar fluctuations were generated. The spectrum is scale-invariant if the cosmological background was rapidly expanding or blue if it was slowly contracting.

The difference in expansion rate between inflation and ekpyrotic/cyclic models also affects the predictions of higher order non-gaussian corrections to the density fluctuation spectrum. The corrections are proportional to the Hubble parameter, and are, thus, exponentially smaller in ekpyrotic/cyclic models than in inflation.

A third difference is that ekpyrotic/cyclic models depend on a single field, the modulus field that determines the distance between branes. Only fluctuations in this one degree of freedom created the initial perturbations. Consequently, perturbations are purely adiabatic. In inflation, various schemes exist for producing an isocurvature component by adding a second field to the reheating process.

Altogether, then, there are at least four ways of distinguishing inflation and ekpyrotic/cyclic models.

- The tensor spectrum is nearly scale-invariant for inflation but strongly blue for ekpyrotic models.
- The higher-order non-gaussian corrections to the scalar spectrum ($\propto H$ when the modes exit the horizon) is exponentially greater in inflation than in ekpyrotic models.
- The primordial ekpyrotic spectrum is purely adiabatic whereas inflation may be adiabatic or a mixture of adiabatic or isocurvature, depending on whether there are one or more fields dominant during inflation and reheating.
- The dark energy is a form of quintessence and unstable in the cyclic theory whereas inflation makes no prediction.

The tensor spectrum, is the most promising since the technology exists to test the natural inflationary prediction.

Myth 10: Stringy effects on the density perturbations are difficult or impossible to observe

Another important difference between inflation and ekpyrotic/cyclic models is the physical interpretation of the density fluctuation amplitude. In inflation, any direct knowledge of conditions near the string or Planck scale are wiped out by the accelerated expansion. The density fluctuation amplitude only depends on the conditions when the perturbations were generated during inflation.

In the ekpyrotic/cyclic models, the perturbations exit the horizon during the contracting phase and, then, propagate through the bounce. The bounce depends explicitly on 5d physics: the warp factor between branes, the velocity of collision, and the Z_2 orbifold structure. The final amplitude of the density perturbations after the bounce depends directly on these features, as well as the conditions during the stage of contraction when the modes exited the horizon. The bounce conditions in the cyclic model factor into the amplitude but do not affect the spectral shape.

From this discussion emerges a profound conclusion. If inflation is correct, then we are blocked from any direct knowledge of the big bang and any other pre-inflationary conditions. If the ekpyrotic solution is correct, then our measurements of the microwave background fluctuation amplitude are *to leading order* direct probes of the big crunch and big bang, including stringy and extra-dimensional physics. Settling the cosmological issue of whether the density fluctuations were produced during a period of expansion or contraction (by searching for tensor fluctuations) will also determine whether physical conditions near the big bang can be probed empirically or not. This raises the stakes and enhances the importance of distinguishing the two scenarios.

I am very grateful to my collaborators who contributed so much to the work described in this paper: N.Turok, J. Khoury, S. Gratton, A. Tolley, L. Boyle, R. Davis, and M. Turner. I also thank A. Kosowsky for useful discussions. This work was supported in part US Department of Energy grant DE-FG02-91ER40671. PJS is currently Keck Distinguished Visiting Professor at the Institute for Advanced Study with support from the Wm. Keck Foundation and the Monell Foundation.

[1] A.H. Guth, Phys. Rev. D **23**, 347 (1981); A.D. Linde, Phys. Lett. B **108**, 389 (1982); A. Albrecht and P.J. Steinhardt, Phys. Rev. Lett. **48**, 1220 (1982).
[2] J. Bardeen, P.J. Steinhardt and M. S. Turner, Phys. Rev. D **28**, 679 (1983); A.H. Guth and S.-Y. Pi, Phys. Rev. Lett. **49**, 1110 (1982); A.A. Starobinskii, Phys. Lett. B **117**, 175 (1982); S.W. Hawking, Phys. Lett. B **115**, 295 (1982).
[3] J. Khoury, B.A. Ovrut, P.J. Steinhardt and N. Turok, Phys. Rev. D **64**, 123522 (2001).
[4] P.J. Steinhardt and N. Turok, Science **296**, 1436 (2002); Phys. Rev. D **65**, 126003 (2002).
[5] J. Khoury, P.J. Steinhardt, N. Turok, astro-ph/0302012.
[6] L. Boyle, P.J. Steinhardt and N. Turok, in preparation.
[7] D. Spergel *et al.*, *Astrophys. J. Suppl.***148**, 175 (2003).
[8] J. Khoury, B.A. Ovrut, P.J. Steinhardt and N. Turok, Phys. Rev. D **66**, 046005 (2002).
[9] S. Gratton, J. Khoury, P.J. Steinhardt and N. Turok, astro- ph/0301395.
[10] U. Seljak and M. Zaldarriaga, *Phys. Rev. Lett.***78**, 2054 (1997).
[11] M. Kamionkowski, A. Kosowsky, and A. Stebbins, *Phys. Rev. Lett.***78**, 2058 (1997).
[12] R.L. Davis, H.M. Hodges, G.F. Smoot, P.J. Steinhardt and M.S. Turner, Phys. Rev. Lett. **69**, 1856 (1992); A. Liddle and D. Lyth, Phys. Lett. B **291**, 391 (1992).
[13] U. Seljak and C.M. Hirata, astro-ph/0310163.
[14] R. Durrer, hep-th/0112026; R. Durrer and F. Vernizzi, Phys. Rev. D **66**, 083503 (2002); C. Cartier, R. Durrer and E.J. Copeland, hep- th/0301198.
[15] R. Brandenberger and F. Finelli, JHEP 0111, 056 (2001); F. Finelli and R. Brandenberger, Phys. Rev. D **65**, 103522 (2002); D. Lyth, Phys. Lett. B **524**, 1 (2002); Phys. Lett. B **526**, 17 (2002); J.C. Hwang, Phys. Rev. D **65**, 063514 (2002); P. Peter and N. Pinto-Neto, Phys. Rev. D **66**, 063509 (2002).
[16] A. Tolley, N. Turok and P.J. Steinhardt, hep-th/0306109.

[17] M. Tegmark, *et al.*, astro-ph/0310723.
[18] L. Wang, V.F. Mukhanov and P.J. Steinhardt, Phys. Lett. B **414**, 18 (1997).
[19] L. Boyle, P.J. Steinhardt and N. Turok, hep-th/0307170.
[20] J. Khoury, P.J. Steinhardt and N. Turok, hep-th/0307132.